**Total variation-energy method**

注1：简介与算法见<ROF+Chambolle.pdf>和<ROFScheme.pdf>

注2：具体过程见[Mathematical Problems in Image Processing.pdf](Mathematical%20Problems%20in%20Image%20Processing.pdf) 第三章p94

注3：关于变分法的具体推导过程，见[欧拉拉格郎日函数.docx](%E6%AC%A7%E6%8B%89%E6%8B%89%E6%A0%BC%E9%83%8E%E6%97%A5%E5%87%BD%E6%95%B0.docx)与笔记。（及<../../partial_differential_equations.pdf>第八章）

**Total variation-a dual method**

(参考[Dual Methods for Total Variation-Based Image Restoration.pdf](Dual%20Methods%20for%20Total%20Variation-Based%20Image%20Restoration.pdf))

**1.Total Variation (TV) Regularization**



[Rudin-Osher-Fatemi 92]



– u need not be differentiable（u不需要可微）

– Discontinuities allowed（可以不连续）

– Derivatives considered in the weak sense

**2. Euler-Lagrange Equation**





Degenerate when |∇u| = 0, so



其中small β > 0

β large Smeared edges.

β small PDE nearly degenerate.

因此通过重写公式去除β，引入对偶变量w.

**3. Dual TV Problem**





Deriving Dual Formulation







Advantages

* Quadratic objective function in w
* No need for β

– u=z+α(∇·w)

Disadvantages

* Constrained optimization problem
* One constraint per pixel

**4. Dual Total Variation Algorithms**

• Primal-Dual Interior Point (Developed by Mulet)

• Relaxation (Coordinate Descent) Methods: Easy to code

– Dual

– Hybrid

* Barrier: Constrained → unconstrained Suggested by Vandenberghe.

**5. Chan–Golub–Mulet’s Primal-Dual Method**（参考[Numerical Methods and Applications in Total Variation Image Restoration.pdf](Numerical%20Methods%20and%20Applications%20in%20Total%20Variation%20Image%20Restoration.pdf)）



它的欧拉-拉格朗日方程为(具体推导见笔记)







总结：Dual and Primal-Dual Methods.（参考[Numerical Methods and Applications in Total Variation Image Restoration.pdf](Numerical%20Methods%20and%20Applications%20in%20Total%20Variation%20Image%20Restoration.pdf) p1073）

**6.TV numerical methods**

除了上述所提的方法外，处理全变分还有如下方法：

1.Chan–Golub–Mulet’s Primal-Dual Method

2. Chambolle’s Dual Method

3. Primal-Dual Hybrid Gradient Method

4. Semi-Smooth Newton’s Method

5. Primal-Dual Active-Set Method

Krishnan的non-negatively constrained CGM，combining the CGM and the PDAS algorithms<../KrishnanLinYip.pdf>

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